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ON AN INTEGRAL EQUATION AND ITS APPLICATION TO PROBLEMS OF THIN DETACHED INCLUSIONS IN ELASTIC BODIES*

B.I. SMETANIN

An integral equation of the first kind, that occurs in certain elasticity theory problems for bodies with detached inclusions /1/, is examined. The structure of its solution is established. The effective solution of this equation is constructed by the asymptotic method of "large λ" /2/. The antiplane problem of the shear of detached bands located in the middle plane of an elastic layer is investigated as an example.

1. We consider the following integral equation:

$$\int_{-1}^{1} q(\xi)\, k\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi f(x) \quad (|x| \leqslant 1, \lambda \in (0, \infty)) \tag{1.1}$$

Here $q(x)$ is a function to be determined, and $k(t)$ and $f(x)$ are given functions. The kernel of this equation has the form

$$k(t) = \int_0^\infty [\Lambda_1(u) \cos ut - \Lambda_2(u) \sin ut] \frac{du}{u} \tag{1.2}$$

The function $\Lambda_n(z)$ $(n = 1, 2)$ in the plane of the complex variable $z = u + iv$ are meromorphic functions that are real for $v = 0$. On the $v = 0$ axis the function $\Lambda_1(u)$ has the single zero $u = 0$ and has no poles, while the function $\Lambda_2(z)$ has neither zeros nor poles on the $v = 0$ axis. As $|u| \to \infty$ the functions $\Lambda_n(u)$ satisfy the condition

$$\Lambda_n(u) \to 1 + O(e^{-\varkappa_n |u|}) \quad (|u| \to \infty;\ \varkappa_n > 0;\ n = 1, 2) \tag{1.3}$$

Lemma 1. For all values of $0 \leqslant |t| < \infty$ the following representation holds:

$$k(t) = -\ln|t| - 0{,}5\pi \operatorname{sgn} t + F(t) \tag{1.4}$$

$$F(t) = \int_0^\infty \{[\Lambda_1(u) - 1] \cos ut + e^{-u} - [\Lambda_2(u) - 1] \sin ut\} \frac{du}{u} \tag{1.5}$$

where $F(w)$ is, as a function of the complex variable $w = t + i\tau$, regular in the strip $|t| < \infty$, $|\tau| < \varkappa_* = \min(\varkappa_1, \varkappa_2)$. For $|t| < \varkappa_*$ the function $F(t)$ can be represented by the absolutely convergent series

$$F(t) = \sum_{n=0}^{\infty} b_n t^n \tag{1.6}$$

To prove (1.4) and (1.5) we must use Eq.(22.29) of /2/ and the integral /3/

$$\int_0^\infty \frac{\sin ut}{u} du = \frac{\pi}{2} \operatorname{sgn} t$$

The regularity of the function $F(t)$ in the band $|\tau| < \varkappa_*$ results from the properties of the functions $\Lambda_n(z)$ and the Theorem *A* presented in Sect.1.4 of /4/. From the regularity of $F(w)$ it follows that $F(t)$ is continuous with all its derivatives for $0 \leqslant |t| < \infty$. Expanding $\cos ut$ and $\sin ut$ in (1.5) in power series, we obtain the representation (1.6) and the coefficients of this expansion in the form

$$b_0 = \int_0^\infty [\Lambda_1(u) - 1 + e^{-u}] \frac{du}{u} \tag{1.7}$$

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$$b_{2n}=\frac{(-1)^n}{(2n)!}\int_0^\infty [\Lambda_1(u)-1]\,u^{2n-1}\,du \quad (n\neq 0)$$

$$b_{2n+1}=\frac{(-1)^n}{(2n+1)!}\int_0^\infty [1-\Lambda_2(u)]\,u^{2n}\,du \quad (n=0,1,\ldots)$$

Since $|t|\leqslant 2/\lambda$, the solution of (1.1), obtained by using the expansion (1.6), will have meaning for at least $\lambda>\lambda_1$, where $\lambda_1=2/\varkappa_*$.

Taking account of (1.4), we convert the integral equation (1.1) to the form

$$\int_{-1}^{1}\left[\ln\frac{\lambda}{|\xi-x|}+b_0-\frac{\pi}{2}\operatorname{sgn}(\xi-x)\right]q(\xi)\,d\xi=\pi f(x)-\int_{-1}^{1}q(\xi)\left[F\left(\frac{\xi-x}{\lambda}\right)-b_0\right]d\xi \quad (|x|\leqslant 1) \qquad (1.8)$$

Furthermore, we consider the integral equation

$$\int_{-1}^{1}\left[\ln\frac{\lambda}{|\xi-x|}+b_0-\frac{\pi}{2}\operatorname{sgn}(\xi-x)\right]q(\xi)\,d\xi=\pi f(x) \quad (|x|\leqslant 1) \qquad (1.9)$$

As will be shown below, the solution of (1.9) is the principal term of the asymptotic of the solution of (1.1) for large values of the parameter λ. By differentiating (1.9) with respect to x we obtain the singular integral equation

$$\pi q(x)+\int_{-1}^{1}\frac{q(\xi)}{\xi-x}\,d\xi=\pi f'(x) \quad (|x|\leqslant 1) \qquad (1.10)$$

The solution of (1.10) has the form /5/

$$q(x)=\frac{Q}{\pi\sqrt{2}X(x)}+\frac{f'(x)}{2}-\frac{1}{2\pi}\int_{-1}^{1}\frac{X(\xi)f'(\xi)}{X(x)(\xi-x)}\,d\xi \qquad (1.11)$$

$$Q=\int_{-1}^{1}q(x)\,dx, \quad X(x)=(1+x)^{3/4}(1-x)^{1/4} \qquad (1.12)$$

The first term on the right side of (1.11) is the solution of the homogeneous equation (1.10), consequently the constant Q is arbitrary.

Formula (1.11) is also the solution of (1.9) under the condition

$$Q=\frac{1}{\sqrt{2}\,(\ln 4\lambda+b_0)}\int_{-1}^{1}\frac{f(x)}{X(-x)}\,dx \qquad (1.13)$$

To obtain (1.13) we must multiply (1.9) by $X^{-1}(-x)$ and integrate with respect to x between the limits -1 and 1 by using (6.7) from Table *A* in /1/.

Furthermore, we need the values of the integral /6/

$$\int_{-1}^{1}\frac{\xi^n X(\xi)}{\xi-x}\,d\xi=\pi x^n X(x)-\pi\sqrt{2}\sum_{m=0}^{n-1}\sum_{j=0}^{m}\frac{(-1)^j(-3/4)_j(-1/4)_{m-j}}{(m-j)!\,j!}x^{n-m-1} \qquad (1.14)$$

$$(z)_m=z(z+1)\ldots(z+m-1), \quad (z)_0=1, \quad n=0,1,\ldots$$

Let us investigate the structure of the solution of the integral Eq.(1.9).

Theorem 1. If the function $f(x)\in H_{n-1}^{\alpha}(-1,1)$, $n\geqslant 0$, $0<\alpha$, then the solution $q(x)$ of the integral Eq.(1.9) has the form

$$q(x)=\omega(x)X^{-1}(x), \quad \omega(x)\in C_n(-1,1) \qquad (1.15)$$

Here $H_n^{\alpha}(-1,1)$ is the space of functions whose n-th derivative satisfies the Hölder condition with index α for $x\in[-1,1]$, and $C_n(-1,1)$ is the space of functions whose n-th derivative is continuous for $x\in[-1,1]$ /7/.

To prove the theorem, we transform (1.11) to the following form by taking (1.14) into account

$$\omega(x)=Q(\pi\sqrt{2})^{-1}+(2\sqrt{2})^{-1}(1+2x)f'(x)-\chi(x) \qquad (1.16)$$

$$\chi(x)=\frac{1}{2\pi}\int_{-1}^{1}\frac{X(\xi)[f'(\xi)-f'(x)]}{\xi-x}\,d\xi \qquad (1.17)$$

Differentiating the integral (1.17) formally n times with respect to x and using (7.4) from /8/, we obtain

$$\chi^{(n)}(x)=\frac{1}{2\pi}\int_{-1}^{1}\frac{X(\xi)\chi_*(x,\xi)}{|\xi-x|^{\beta}}d\xi \quad (1-\alpha<\beta<1) \tag{1.18}$$

The function $\chi_*(x,\xi)$ satisfies the Hölder condition for $|x|\leqslant 1$, $|\xi|\leqslant 1$. From the boundedness of the function $X(\xi)\chi_*(x,\xi)$ for $|x|\leqslant 1, |\xi|\leqslant 1$ the uniform convergence, with respect to x of the integral (1.18) follows. Differentiation under the sign of this integral is thereby also given a foundation. Therefore, $\chi^{(n)}(x)$ is a continuous function and the thoerem is proved.

Theorem 2. If the function $f(x)\in H^{\alpha}_{n+1}(-1,1)$, $n\geqslant 0$, $^1/_4<\alpha$ and the following relationship is satisfied:

$$Q-\frac{1}{\sqrt{2}}\int_{-1}^{1}\frac{f'(\xi)}{Y(\xi)}d\xi=0 \tag{1.19}$$

then the solution of the integral equation (1.9) has the form

$$q(x)=\psi(x)Y(x),\quad \psi(x)\in C_n(-1,1) \tag{1.20}$$

To prove the theorem, taking into account the value of the integral

$$\int_{-1}^{1}\frac{d\xi}{Y(\xi)(\xi-x)}=\pi\left[\frac{1}{Y(x)}-\sqrt{2}\right],\quad Y(x)=\left(\frac{1+x}{1-x}\right)^{1/4} \tag{1.21}$$

and relationship (1.19), relationship (1.11) must be converted to the form

$$q(x)=\frac{Y(x)}{\sqrt{2}}f'(x)-\frac{Y(x)}{2\pi}\int_{-1}^{1}\frac{f'(\xi)-f'(x)}{Y(\xi)(\xi-x)}d\xi \tag{1.22}$$

Furthermore, the proof is analogous to the proof of Theorem 1. We note that condition (1.19) is equivalent to the condition $\omega(-1)=0$.

Theorem 3. If the function $f(x)\in H^{\alpha}_{n+1}(-1,1)$, $n\geqslant 0$, $0<\alpha\leqslant 1$, and the solution of the integral equations (1.1) exists in $L_p(-1,1)$, $p>1$, then the solution $q(x)$ of this equation has the following form for all $\lambda\in(0,\infty)$

$$q(x)=\Omega(x)X^{-1}(x),\quad \Omega(x)\in C_n(-1,1) \tag{1.23}$$

Here $L_p(-1,1)$ is the space of absolutely summable functions with power p for $x\in[-1,1]$ /7/.

Theorem 4. If the function $f(x)\in H^{\alpha}_{n+1}(-1,1)$, $n\geqslant 0$, $^1/_4<\alpha\leqslant 1$, and the solution of the integral Eq.(1.1) exists in $L_p(-1,1)$, $p>1$, and the condition

$$\Omega(-1)=0 \tag{1.24}$$

is satisfied, then the solution $q(x)$ of this equation has the following form for $\lambda\in(0,\infty)$:

$$q(x)=\Psi(x)Y(x),\quad \Psi(x)\in C_n(-1,1) \tag{1.25}$$

The proof of Theorems 3 and 4 is constructed by using Theorems 1 and 2 and Lemma 1, and is analogous to the proof of Theorems 24.1 and 24.2 in /2/.

Lemma 2. If the function $f(x)\in H^{\alpha}_{n+1}(-1,1)$, $0<\alpha\leqslant 1$, then any solution of the integral Eqs.(1.1) or (1.8) from the class $L_p(-1,1)$, $p>1$ is also a solution of the integral equation

$$q(x)=\frac{Q}{\pi\sqrt{2}X(x)}+\frac{f'(x)}{2}-\frac{1}{2\pi}\int_{-1}^{1}\frac{X(\xi)f'(\xi)}{X(x)(\xi-x)}d\xi+ \tag{1.26}$$

$$\frac{1}{2\pi^2\lambda X(x)}\int_{-1}^{1}q(\xi)M(x,\xi)d\xi\quad(|x|\leqslant 1)$$

$$M(x,\xi)=\pi X(x)F'\left(\frac{\xi-x}{\lambda}\right)-\int_{-1}^{1}\frac{X(\eta)}{\eta-x}F'\left(\frac{\xi-\eta}{\lambda}\right)d\eta \tag{1.27}$$

$$Q=\frac{1}{\sqrt{2}\ln 4\lambda}\left[\int_{-1}^{1}\frac{f(x)\,dx}{X(-x)}-\frac{1}{\pi}\int_{-1}^{1}\frac{dx}{X(-x)}\int_{-1}^{1}q(\xi)F\left(\frac{\xi-x}{\lambda}\right)d\xi\right] \tag{1.28}$$

and conversely. The integral on the right side of (1.8) is a continuous function will all its derivatives with respect to x for $x\in[-1,1]$ and $\lambda\in(0,\infty)$ because of the properties of the function $F(t)$ and the condition $q(x)\in L_p(-1,1)$, $p>1$. Taking account of (1.13) when using the inversion formula (1.11), we obtain (1.26)-(1.28). The possibility of changing the order of integration in the double integral in (1.26)-(1.27) is based on utilizing the properties of the functions $q(x)$ and $F(t)$ and the lemma from Sect.7 in /5/. The converse assertion of the lemma results from the possibility of converting from (1.26)–(1.28) to (1.1) for $q(x) \in L_p(-1, 1)$, $p > 1$.

If $f(x) \in H_1^\alpha(-1, 1)$, $0 < \alpha \leqslant 1$, then by virtue of Theorem 3 the solution of the integral Eq.(1.8) in the class $q(x) \in L_p(-1, 1)$, $p > 1$ can be sought in the from (1.23), where $\Omega(x) \in C(-1, 1)$. In this case, (1.26) can be represented in the form

$$\Omega = \Omega^0 + A(\Omega) \tag{1.29}$$

$$\Omega^0(x) = \frac{Q}{\pi\sqrt{2}} + \frac{f'(x)}{2} X(x) - \frac{1}{2\pi}\int_{-1}^{1} \frac{X(\xi) f'(\xi)}{\xi - x} d\xi \tag{1.30}$$

$$A(\Omega) = \frac{1}{2\pi^2\lambda}\int_{-1}^{1} \frac{\Omega(\xi)}{X(\xi)} M(x, \xi) d\xi \tag{1.31}$$

Lemma 3. The operator A defined by (1.31), acts in the space $C(-1, 1)$.

To prove the lemma, by taking account of (1.14) the function $M(x, \xi)$ should be converted to the form

$$M(x, \xi) = \frac{\pi}{\sqrt{2}}(1 + 2x) F'\left(\frac{\xi - x}{\lambda}\right) - I(x, \xi) \tag{1.32}$$

$$I(x, \xi) = \int_{-1}^{1} \left[F'\left(\frac{\xi - \eta}{\lambda}\right) - F'\left(\frac{\xi - x}{\lambda}\right)\right] \frac{d\eta}{X^{-1}(\eta)(\eta - x)}$$

The proof of the lemma is furthermore analogous to the proof of Lemma 25.2 in /2/.

Theorem 5. Let the function $f(x) \in H_1^\alpha(-1, 1)$, $0 < \alpha \leqslant 1$ and let the following inequality hold

$$\lambda > \lambda_2 = \frac{3}{4}\left[D_1 + \sqrt{D_1^2 + {}^2/_3 D_2}\right] \tag{1.33}$$

$$D_1 = \max |F'(t)|, \quad D_2 = \max |F''(t)|, \quad t \in [0, \infty]$$

In this case the solution of integral Eq.(1.29) in the class $C(-1, 1)$ exists, is unique, and can be obtained by usccessive approximations according to the scheme

$$\Omega^n(x) = \Omega^0(x) + A(\Omega^{n-1}) \tag{1.34}$$

To prove the theorem, we estimate $|M(x, \xi)|$. From (1.32) we obtain

$$|M(x, \xi)| \leqslant \frac{3\pi}{\sqrt{2}} \max |F'(t)| + |I(x, \xi)| < \frac{3\pi}{\sqrt{2}}\left(D_1 + \frac{1}{4\lambda} D_2\right) \tag{1.35}$$

Taking account of (1.35) we determine from (1.31)

$$\| A(\Omega) \|_C = \frac{1}{2\pi^2\lambda} \max_{|x| \leqslant 1} \left| \int_{-1}^{1} \frac{\Omega(\xi)}{X(\xi)} M(x, \xi) d\xi \right| < \frac{3}{2\lambda} \| \Omega \|_C \left(D_1 + \frac{1}{4\lambda} D_2\right) \tag{1.36}$$

from which it follows that the operator A is contractive in $C(-1, 1)$ when condition (1.33) is satisfied by virtue of the Banach "fixed-point" principle.

We examine a more convenient means for constructing the approximate solution of the integral Eq.(1.26) for large values of the parameter λ. We will seek the solution of this equation in the form /2/

$$q(x) = \sum_{n=0}^{\infty} q_n(x) \lambda^{-n} \tag{1.37}$$

Substituting (1.37) and (1.6) into (1.26) and (1.27), using (1.14) and then equating terms of identical powers of λ on the left and right sides of the equation obtained, we arrive at the following infinite system of equations for the sequential determination of the function $q_n(x)$:

$$q_0(x) = \frac{Q}{\pi\sqrt{2}X(x)} + \frac{f'(x)}{2} - \frac{1}{2\pi}\int_{-1}^{1} \frac{X(\xi) f'(\xi)}{X(x)(\xi - x)} d\xi \tag{1.38}$$

$$q_1(x) = \frac{(1 + 2x) b_1}{2\sqrt{2}\pi X(x)} \int_{-1}^{1} q_0(\xi) d\xi$$

$$q_2(x) = \frac{b_2}{\sqrt{2}\pi X(x)} \int_{-1}^{1} q_0(\xi)\left[(1 + 2x)(\xi - x) + \frac{3}{4}\right] d\xi + \frac{(1 + 2x) b_1}{2\sqrt{2}\pi X(x)} \int_{-1}^{1} q_1(\xi) d\xi$$

etc. It can be shown that the function $q_n(x)$ should satisfy the condition

$$\int_{-1}^{1} q_n(x)\,dx = \begin{cases} Q, & n=0 \\ 0, & n\neq 0 \end{cases} \tag{1.39}$$

Condition (1.39) can be used to check on the correctness of the determination of the functions $q_n(x)$ and to facilitate finding them by means of (1.38). Cutting off the process of determining the functions $q_n(x)$ we obtain the approximate solution of the integral Eq.(1.1) in the form

$$q(x) = \sum_{n=0}^{N} q_n(x)\lambda^{-n} + O(\lambda^{-N-1}) \tag{1.40}$$

Having found the functions $q_n(x)$, the constant Q is determined from the following equation that results from (1.28)

$$Q(\ln 4\lambda + b_0) = \frac{1}{\sqrt{2}}\int_{-1}^{1}\frac{f(x)\,dx}{X(-x)} - \frac{b_1}{\lambda}\int_{-1}^{1} q_0(\xi)\left(\xi-\frac{1}{2}\right)d\xi - \tag{1.41}$$
$$\frac{1}{\lambda^2}\left[b_1\int_{-1}^{1} q_1(\xi)\,\xi\,d\xi + b_2\int_{-1}^{1} q_0(\xi)\left(\xi^2-\xi+\frac{5}{8}\right)d\xi\right] + O(\lambda^{-3})$$

Carrying out calculations by means of (1.38) and (1.41) for the special case of $f(x) = f_* = \text{const}$, we obtain

$$q(x) = \frac{Q}{\pi\sqrt{2}X(x)}\left\{1+\left(x+\frac{1}{2}\right)\frac{b_1}{\lambda} - \left(x^2+x-\frac{1}{8}\right)\frac{2b_2}{\lambda^2} - \right. \tag{1.42}$$
$$\left[\left(\frac{1}{4}x+\frac{1}{8}\right)b_2 - \left(x^3+\frac{3}{2}x^2+\frac{3}{4}x-\frac{1}{8}\right)b_3\right]\frac{3}{\lambda^3} -$$
$$\left[b_2^2\left(x+\frac{1}{2}\right) - 3b_3\left(\frac{3}{4}x^2+\frac{1}{2}x-\frac{7}{32}\right)+\right.$$
$$\left.\left. b_4\left(4x^4+8x^3+9x^2+3x-\frac{87}{32}\right)\right]\frac{1}{\lambda^4} + O(\lambda^{-5})\right\}$$

$$Q = \pi f_*\left\{\ln 4\lambda + b_0 - \frac{b_1}{\lambda} + \left(\frac{3}{8}b_1^2+\frac{7}{4}b_2\right)\frac{1}{\lambda^2} - \right. \tag{1.43}$$
$$\left.\left(b_1b_2+\frac{11}{4}b_3\right)\frac{1}{\lambda^3} - \left(\frac{9}{32}b_1b_2 - \frac{27}{16}b_1b_3 - \frac{31}{64}b_2^2 - \frac{329}{64}b_4\right)\frac{1}{\lambda^4} + O(\lambda^{-5})\right\}^{-1}$$

The solution obtained for the integral Eq.(1.1) can be used for $\lambda > \lambda_3 = \max(\lambda_1, \lambda_2)$.

2. As an example, we consider the following problem. Let a domain occupied by an elastic medium be the infinite layer $|y| \leqslant h$, $|x| < \infty$, $|z| < \infty$. For $|x| \leqslant a$, $|z| < \infty$ in the $y=0$ plane, there is a thin stiff strip. The upper face of the strip is attached to the elastic medium while the lower face is detached from the medium. Under the action of a force T (referred to unit length of the strip), the strip shifts an amount ε in the direction of the z-axis. We shall also consider that the lower face of the elastic layer is attached to an undeformable foundation while the upper face is load-free.

This problem is reduced to the solution of the integral Eqs.(1.1), (1.2) by the method of integral transforms. Here

$$f(x) = f_* = \frac{2\varepsilon}{a}, \quad \Lambda_1(u) = \operatorname{th} 2u, \quad \Lambda_2(u) = \frac{\operatorname{th} 2u}{\operatorname{th} u}, \quad \lambda = \frac{h}{a} \tag{2.1}$$

$$\tau(x) = \frac{\mu}{2}\left[q\left(\frac{x}{a}\right)+q\left(-\frac{x}{a}\right)\right], \qquad T = \int_{-a}^{a}\tau(x)\,dx = a\mu Q \tag{2.2}$$
$$w_x'(x,-0) = \frac{1}{2}\left[q\left(\frac{x}{a}\right)-q\left(-\frac{x}{a}\right)\right] \qquad (|x| \leqslant a)$$

Here μ is the shear modulus, $\tau(x)$ is a function characterizing the shear stress distribution in the domain of strip contact with the elastic medium, and $w(x, y)$ is the projection of the displacement vector on the z-axis. Evaluating the integrals in (1.7) by taking account of the values of the functions $\Lambda_n(u)$, we obtain

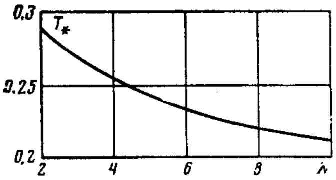
$$b_0 = \ln\frac{8}{\pi}, \quad b_{2n} = \frac{\pi^{2n}(2^{2n-1}-1)B_{2n}}{4^{2n}n(2n)!} \quad (n=1,2,\ldots) \tag{2.3}$$
$$b_{2n+1} = -\frac{\pi^{2n+1}E_{2n}}{4^{2n+1}(2n-1)!} \quad (n=0,1,\ldots)$$

The B_m in (2.3) are Bernoulli numbers, and the E_m are Euler numbers /3/. From (2.1) and (1.5) we determine $\varkappa_1 = 4$, $\varkappa_2 = 2$, $D_1 = 1$, $D_2 = 1/2$. Therefore $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 1.616$. In practice, (1.42) and (1.43) can be used for $\lambda \geqslant 2$ for the case under consideration.

The dependence of the quantity $T_* = T(2\pi\mu\varepsilon)^{-1}$ on the parameter λ obtained by using (1.43) and (2.1)–(2.3), is represented in the figure. For $10 \leqslant \lambda \leqslant \infty$

$$T_* = [\ln(32\lambda\pi^{-1})]^{-1} \tag{2.4}$$

can be used with an error not exceeding 2%.



As $\lambda \to \infty$, the dependence between the force T and the magnitude ε of the strip displacement is impossible to determine.

The solution of the integral Eq.(1.1) can also be constructed by the method of orthogonal polynomials. It is here convenient to use the spectral relation (6.7) from Table A in /1/.

We note that the function characterizing the contact stress distribution in axisymmetric problems about thin detached inclusions in elastic bodies is expressed in terms of the solution of an equation of the form (1.1) by using a certain integral operator. The condition that this function belongs to the space $L_p, p > 1$ results in the need to construct a solution of (1.1) in the form (1.25).

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